Ship resistance in navigation canals

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Abstract. This paper describes a mathematical model of ship resistance in constricted navigation channels. The model is based on estimation of power required for a particular ship to move at a given speed in a canal of given cross-section, and on the vertical sinkage of the ship which is dependent of ship's speed. Under certain assumptions, the sinkage can be calculated by solving the equation of balance of horizontal forces acting on the fluid within the control volume around the ship. The proposed model is validated by resistance measurements performed for a particular prototype ship. An attempt is made to generalize the results of calculations.

Key words: ship resistance, navigation canals

1 Introduction

The term "ship resistance" refers to the intensity of force opposing the ship's movement. Naval architects study ship resistance for design purposes, while civil engineers study this phenomenon in order to optimize the cross-section of a navigation canal (Fig. 1).

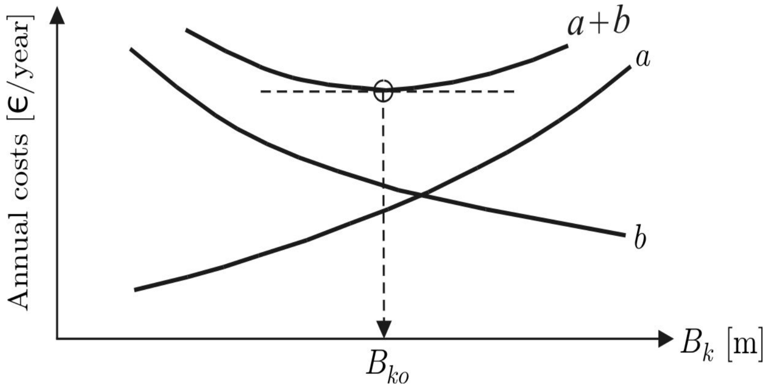

Figure 1: *Annual costs depending on the navigation canal's width* (B_k)*: a – the annual cost of the canal, b – the annual fleet operation cost, comprising the annual cost of fuel, which is proportional to the ship resistance; the optimal canal width* (B_{ko}) *is derived by superposition of annual costs.*

Ship resistance is a long time topic of research. In addition to the literature on general ship hydrodynamics (for instance, [1, 6]), there are numerous publications dedicated to certain ship types and their operational speeds (for instance, [3, 4, 11]), papers dealing with the investigation of the flow field around the ship and ship-induced waves(for instance, [2, 10]), and finally, papers considering problems of mathematical modelling of ship resistance (for instance, [**?**]). In majority of these sources, ship resistance is considered in waterways of unlimited depth and width, and only a relatively small number of papers deals with resistance of ships on restricted water.

As the distribution of normal and shear stresses over the submerged surface of the ship is not known, it is practically impossible to calculate the exact force opposing ship's motion. An approximate value can be determined, as is well known, by assuming that the total resistance R_u is the sum of the frictional resistance denoted by R_t, and the "shape resistance", due to the differential pressure and waves, denoted by R_o:

$$R_u = R_t + R_o. \tag{1}$$

The frictional resistance depends on the submerged surface area of hull, its absolute roughness, and the flow velocity along the submerged surface. Resistance due to differential pressure on bow and stern and resistance due to the ship-generated waves (Fig. 2) are difficult to separate, and are thus treated together as the "shape resistance".

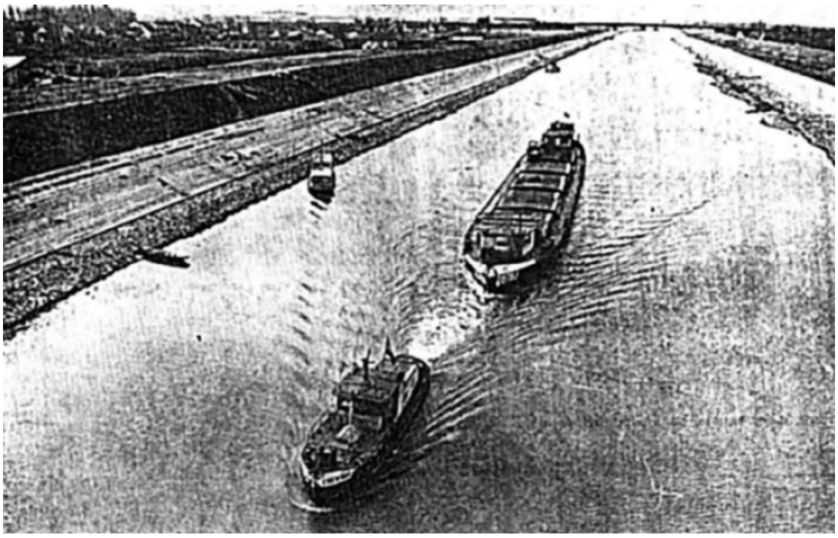

Figure 2: *The navigation canal near the city of Novi Sad in Serbia* [3].

Forces on the right-hand side of equation (1) have a general form:

$$R = \frac{1}{2} \cdot \rho \cdot C \cdot A \cdot V^{\alpha}, \tag{2}$$

where: C and α are empirical coefficients, ρ is water density, A and V are characteristic area and velocity, respectively. A number of empirical expressions of type (2), found in literature or used in practice, are not dimensionally homogeneous ($\alpha \neq 2$), and therefore lack generality. In order to reduce uncertainties, an original, hydraulically-based method for estimating the ship resistance under sub-critical speeds in navigation canals is developed, with the objective to provide a general, relatively simple computational model for practical applications.

2 Mathematical model

Assumptions. Assume a ship is navigating at a constant speed V_{pl} in respect to the banks of a canal, schematically shown in Figures 3 and 4.

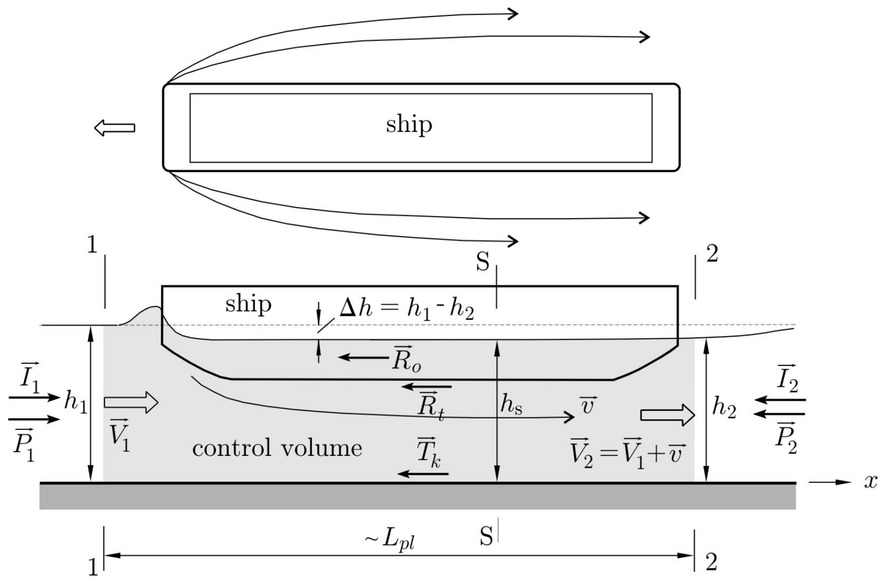

Figure 3: *Longitudinal profile of a navigation canal.*

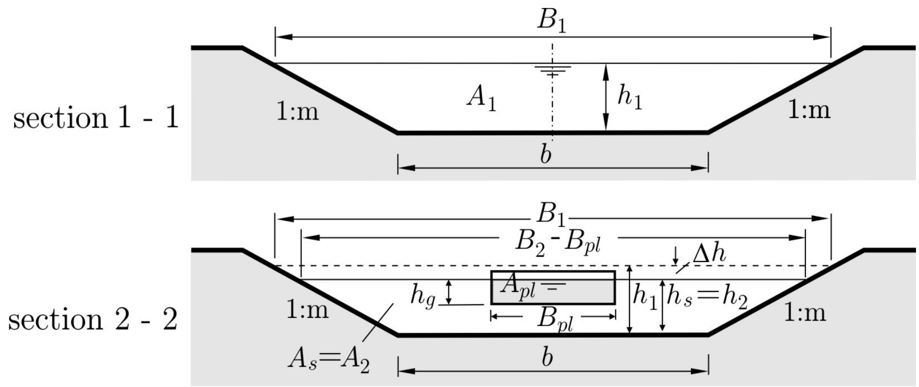

Figure 4: *Cross-sections of a navigation canal relevant for calculation of ship resistance.*

The following additional assumptions are made:

(i) the canal is prismatic, with trapezoid cross-section, and negligible longitudinal bed slope;
(ii) the ship is stationary, while the flow in the canal is steady and uniform, with velocity in the undisturbed cross-section 1-1 equal to the ship's speed : $V_1 = V_{pl}$. If there exists an initial flow in the canal with velocity v_o, calculation is based on the relative velocity:

$$V_1 = \begin{cases} V_{pl} + v_o & - \quad \text{upstream} \quad \text{navigation;} \\ V_{pl} - v_o & - \quad \text{downstream} \quad \text{navigation;} \end{cases} \tag{3}$$

(iii) the ship is far away from banks, so that effect of banks can be neglected;
(iv) the ship's longitudinal axis is horizontal;
(v) sinkage is constant along the ship: Δh = const (Fig. 3).

Basic equations. In the conventional approach, the hydraulic variables relevant for ship resistance estimation – the mean velocity V_s, and depth h_s in the constricted section S-S shown in Figures 3 and 4, can be determined by solving the system of equations representing the mass and energy conservation for an ideal fluid:

$$V_1 \cdot A_1 = V_s \cdot A_s \tag{4}$$

$$h_1 + \frac{V_1^2}{2g} = h_s + \frac{V_s^2}{2g}. \tag{5}$$

Section S-S can be located anywhere along the constriction, and when moved at the downstream boundary of the control volume, section S-S becomes section 2-2 in Fig. 3.

Long time ago, an original idea has been presented in literature [7] that ship resistance can be indirectly determined from the net power necessary for the ship to maintain its speed V_1:

$$P_b = \rho \cdot g \cdot V_1 \cdot A_{pl} \cdot \Delta h \quad [\text{kW}], \tag{6}$$

where: $A_{pl} = Bpl \cdot h_g$ – the submerged area of ship's hull (Fig. 4), $\Delta h = h_1 - h_s$ – ship's sinkage, and g – gravitational acceleration. Expression (6), more intuitive than theoretical in origin, has been formulated by analogy with the expression for power of a pump, whereby the quantity $V_1 \cdot A_{pl}$ represents the discharge which the ship, by its movement "pumps" out of the induced depression (Δh), back up to the normal level in the canal. In narrow navigation canals, or constricted waterways, the shape resistance due to differential pressure and waves:

$$R_o = \frac{P_b}{V_1} \quad [\text{kN}] \tag{7}$$

is usually dominant over the frictional resistance, and the expression (7) can be used to approximate the total resistance. In spite of the fact that some measurements have confirmed validity of the pump analogy [7], this approach has never gained a wider application in practice.

In the new approach presented here, the pump analogy is adopted as an excellent example of intuitive engineering reasoning, together with a hydraulically more exact method for calculating ship's sinkage Δh. This means that instead of equation (5) for ideal fluid, a more complex model is developed, taking into account all phenomena affecting the value of the variable Δh.

(a) *Return flow*. The mean cross-sectional velocity of the ship-generated return flow (v) can be determined from the continuity equation, written for cross-sections 1-1 and 2-2 in Fig. 3:

$$V_1 (A_1 - A_2) = v \cdot A_2. \tag{8}$$

This expression simply states that the discharge induced by ship's movement is equal to the discharge of the return flow. Considering that, according to Fig. 4, $A_1 - A_2 = Apl + \bar{B} \cdot \Delta h$, where the mean water surface width is: $\bar{B} = 0,5\,(B_1 + B_2 + B_{pl})$, the return flow velocity is:

$$v = V_1 \frac{A_{pl} + \bar{B} \cdot \Delta h}{A_2}, \tag{9}$$

and velocity in constricted section is: $V_2 = V_1 + v$. The ship's velocity V_1, the return flow velocity v, and the sinkage Δh are mutually dependent quantities; an increase of velocity V_1 results in an increase of velocities v and V_2 and the sinkage Δh, but a decrease in water depth h_2. (Theoretically the lower bound of depth h_2 is the critical depth).

(b) *Balance of forces*. Following the d'Alembert's principle, by introducing the inertial forces, the problem of fluid dynamics becomes a simple problem of balance of forces which act on the fluid inside the control volume specified in Fig. 3:

$$I_1 - I_2 + P_1 - P_2 - T_k - R_t - R_o = 0. \tag{10}$$

In this equation, I_s and I_s are inertial forces, P_1 and P_2 are pressure forces, T_k is the frictional force over the wetted surface of the canal, R_t is the frictional force over the wetted hull surface, and R_o is the force due to differential pressure and waves (shape resistance). These forces are defined as follows:

$$\begin{aligned} I_1 - I_2 &= \rho\, V_1^2\, A_1 - \rho\, V_2^2\, A_2 && (11)\\ P_1 - P_2 &= \frac{1}{2}\rho\, g\, b\,(h_1^2 - h_2^2) + \frac{1}{3}\rho\, g\,\mathrm{m}\,(h_1^3 - h_2^3) && (12)\\ T_k &= \frac{1}{2}\,\rho\, C_{\tau k}\,(O_2 - B_{pl} - 2\,h_g)\, L_{pl}\, v^2 && (13)\\ R_t &= \frac{1}{2}\,\rho\, C_{\tau b}\, A_o\, V_2^2 && (14)\\ R_o &= \frac{1}{2}\,\rho\, C_o\, A_{pl}\, V_1^2, && (15) \end{aligned}$$

where: b – canal bottom width, m – slope of the banks, O – canal wetted perimeter, $C_{\tau k}$ – coefficient of friction of the canal's surface, $C_{\tau b}$ – coefficient of friction of the hull surface, Ω_{pl} – area of the submerged hull, C_o – coefficient of shape resistance, L_{pl} – length of ship, B_{pl} – width of ship, and h_g – ship draught.

Numerical solution. The mathematical model described by system of equations (11)–(15) is a model with three parameters: $C_{\tau k}$, $C_{\tau b}$, and C_o. The problem is that the values of those parameters depend on unknown velocities, and thus, need to be determined as a part of the overall solution. This means that the problem needs to be solved *iteratively*, until certain conditions are satisfied.

Value of the coefficient of friction for the canal can be calculated in each iteration by using some adopted constant value of the Manning's coefficient n, and a current value of the hydraulic radius $R_{2k} = A_2/(O_2 - B_{pl} - 2\,h_g)$:

$$C_{\tau k} = 2\,g \cdot n^2 / R_{2k}^{1/3}. \tag{16}$$

Value of the coefficient of friction for the ship's hull can be calculated in each iteration by the dimensionally homogeneous ITTC formula [3, 11]:

$$C_{\tau\, ittc} = 0.075\,(\log \mathrm{Re} - 2)^{-2}, \tag{17}$$

where Re=$V_2\, L_{pl}/\nu$ is the Reynolds number, and ν is the coefficient of kinematic fluid viscosity [m^2/s].

Value of the coefficient of shape resistance C_o is updated in each iteration by satisfying balance of forces, until the condition that the difference of C_o values in two successive iterations falls below certain small tolerance.

The computational algorithm:

1. The initial value of $C_{\tau k}$ is calculated from equation (16), assuming that: $A_2 = A_1$, and $O_2 = O_1$. The initial value of $C_{\tau b}$ is calculated using the velocity V_1. The initial value of C_o is set using data from literature for a similar ship (for instance, $C_o = 0,1$).
2. For the given navigation speed V_1, intensity of all forces are calculated, and the system of equations (9)–(10) is solved for unknown variables: v (or V_2) and Δh (or h_2). (Some iterative method for numerical solution of non-linear algebraic equations must be used, for instance the method of interval halving).
3. Using equations (6) and (7), the net power P_b, and the total ship resistance R_u are calculated.
4. A new value of the shape resistance is estimated: $R'_o = R_u - R_t$, and the difference from the previous iteration is calculated: $\delta_o = |R'_o - R_o|$.
5. If difference δ_o is small enough (for instance, 0,1 kN), and if the sum of forces (10) is close to zero ($\pm 1 \times 10^{-3}$), the iterative procedure is stopped, and values in the current iteration represent the final solution.
6. If the above conditions are not satisfied, the values $C_{\tau k}$ and $C_{\tau b}$ are corrected using the last (current) velocity and depth values in cross-section 2-2, while the value of C_o is corrected according to the expression: $C'_o = R'_o/(\rho/2\, A_{pl}\, V_1^2)$. Calculation returns to step 2, and the new iteration is initiated.

The results of calculation show that after 2 iterations at most, values $C_{\tau k}$ and $C_{\tau b}$ change so insignificantly that practically become constant, and that in the iterative cycle, only values of the coefficient C_o and the corresponding force $R_o = R_u - R_t$, are subject to correction.

3 Model validation

The proposed model has been validated by field measurement data, undertaken for a particular case study [3]. Resistance was measured for a prototype cargo ship with dimensions $L_{pl}/B_{pl}/h_g$=72/10/2,1 m, towed in the navigation canal near the city of Novi Sad in Serbia (Fig. 2). The canal is of trapezoid cross-section, with the bottom width of 29 m, water surface width 47 m, depth 3 m, and the side slopes 1:3. The towing experiments were performed on a 5 km long reach, between a ship lock and the canal junction with the Danube river. During experiments, there was no traffic in the canal and the ship lock was not operating, thus it was reasonable to assume that the flow velocity in the canal was negligible. At this time, the ship draught was 1,9 m, capacity 940 t, and the coefficient of navigation A_1/A_{pl} = 6,23. Towing speeds were 5 to 8,5 km/h.

Results of calculation, and their comparison with measurements, are shown in Fig. 5. It can be remarked that in this particular case, the conventional model (based on energy equation) and the proposed model (based on balance of forces) overestimate the total resistance in respect to measurements. Only results obtained by the proposed model are of interest here. For ship speeds less than 6,5 km/h, differences between calculated and measured values are less than 20%. For the speed of 8 km/h, difference becomes significant (34%). By tuning the values of the input data and the model parameters, better agreement of calculation and measurements could have been achieved, but this was intentionally not done, because the accuracy of measurement was not defined, and some conditions under which measurements were carried out (the initial flow velocity in the canal, direction of navigation, etc.), were not specified in publication [3]. Therefore, this validation served only to show that results of calculation are reasonably close to the results of field measurements, proving the model's applicability in practice. It is clear that further validation is needed for definite conclusions.

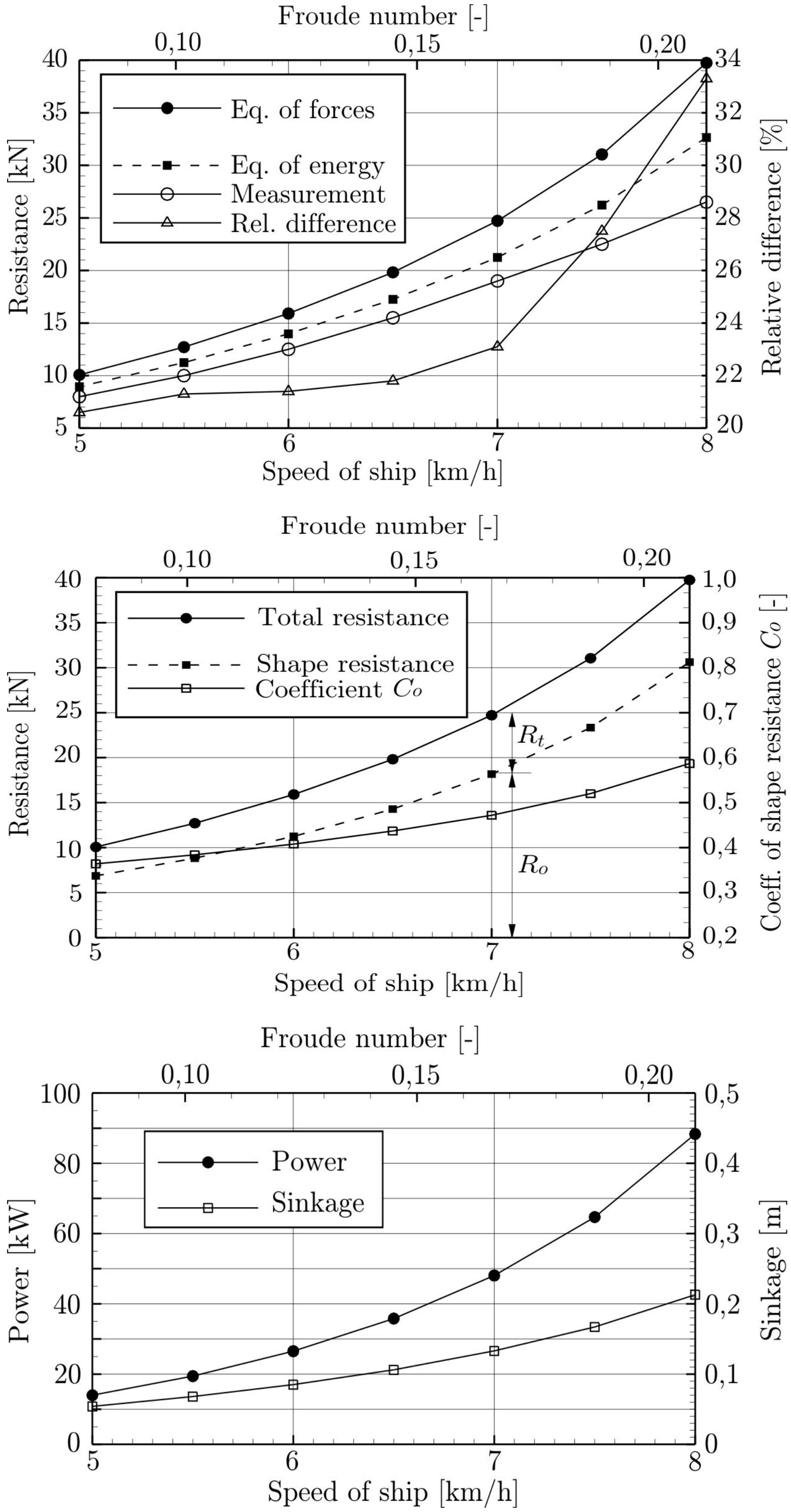

Figure 5: *Results of calculation* (*Froude number refers to the cross-section* 1-1 *upstream from the ship*).

4 Generalization of results

By repeating calculation for a range of input values, a diagram shown in Fig. 6 is produced, in an attempt to generalize results. A parametric relationship is established between the ship resistance (expressed in respect to the ship's weight displacement W_{pl}, in non-dimensional form: R_u/W_{pl}), the Froude number (Fr=$V_{pl}/\sqrt{g\,h}$) , and the parameter of waterway constriction – the "coefficient of navigation" (the ratio between the cross-section of the canal and the submerged cross-section of the ship: A/A_{pl}).

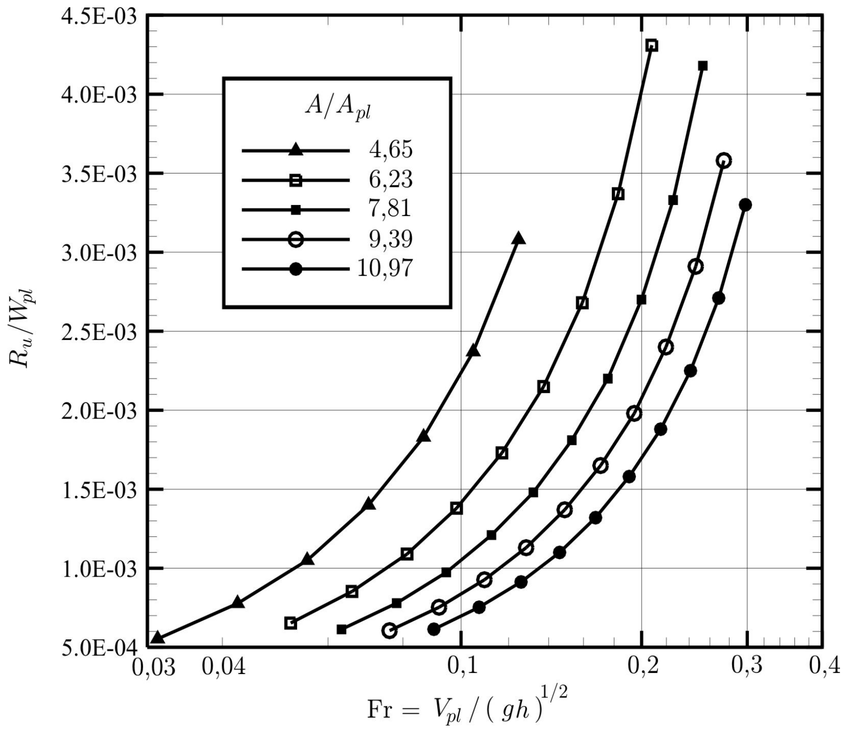

Figure 6: *Ship resistance in navigation canals.*

Results presented in Fig. 6 confirm the well-known fact that the ship resistance decreases as the canal's cross-section increases. It can also be remarked that in the range of values A/A_{pl}=5–11, the maximum values of the Froude number are in the range 0,12–0,30. The practical value of non-dimensional relationships, such as the one shown in Fig. 6, is that hydraulic engineers can quickly make a vaporization of a number of design alternatives for a navigation canal.

5 Conclusion

The analogy between the power of pump and the power of ship yields the possibility of relatively simple estimation of ship resistance in navigation canals, or constricted waterways. A computational procedure is developed, based on iterative solution of equation of balance of forces. The proposed model takes into account all physically relevant factors of ship's navigation in constricted environment, and is for this reason, superior to the conventional approach of ship resistance estimation, including various empirical methods. Comparison with field measurements in one particular case study shows that the proposed model yields reasonably good results, yet more validation is needed for definite conclusion. An attempt to generalize results obtained by this model was made with the purpose to aid hydraulic engineers in design of navigation canals.

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