

(писмени део одржан 11.06.2021.)

Diagram of a frame structure. The structure consists of a horizontal beam of length 8,0 and a vertical column of height 3,0. A distributed load of 5 is applied vertically downwards along the entire length of the beam. A point load of 10 is applied at the free end of the beam, acting at an angle of 15 degrees to the horizontal. The beam is supported by a fixed support at the right end. The coordinate system is defined with the origin at the fixed support, the x-axis along the beam, and the y-axis vertical. The z-axis is perpendicular to the xy-plane.

A 3D coordinate system with axes X, Y, and Z. The Z-axis is vertical, the Y-axis is horizontal to the right, and the X-axis is diagonal down and to the left. The origin is at the intersection of the axes. The following forces are shown:

- F_1 : A force of magnitude $2F$ acting downwards along the negative Z-axis, starting from the point $(-3L, 0, 0)$ on the Y-axis.
- F_2 : A force of magnitude $6F$ acting upwards along the positive Z-axis, starting from the point $(0, 2L, 0)$ on the Y-axis.
- F_3 : A force of magnitude $10F$ acting upwards along the positive Z-axis, starting from the point $(0, 4L, 0)$ on the Y-axis.
- F_4 : A force of magnitude $8F$ acting downwards along the negative Z-axis, starting from the point $(0, 0, 4L)$ on the X-axis.

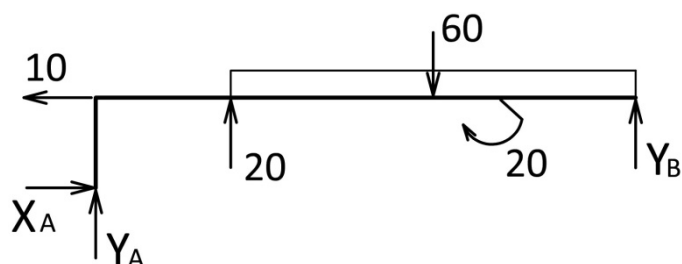
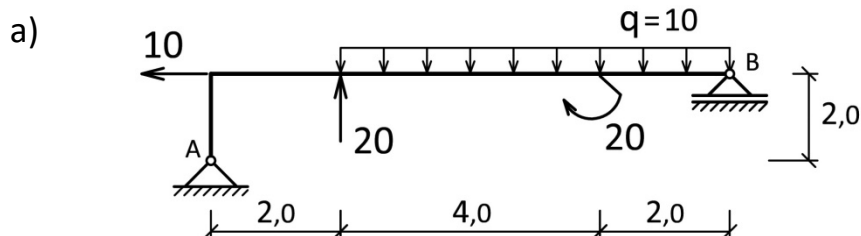
The points where the forces act are marked with dashed lines extending to the axes: $-3L$ on the Y-axis, $2L$ and $4L$ on the Y-axis, and $4L$ on the X-axis.

Напомена: У свим задацима димензије за дужине и силе су: m, kN

ГРАЂЕВИНСКИ ФАКУЛТЕТ УНИВЕРЗИТЕТА У БЕОГРАДУ
 Други (теоријски) део испита из **ТЕХНИЧКЕ МЕХАНИКЕ 1**
 (писмени део одржан 11.06.2021.)

- Р Е Ш Е Њ А -

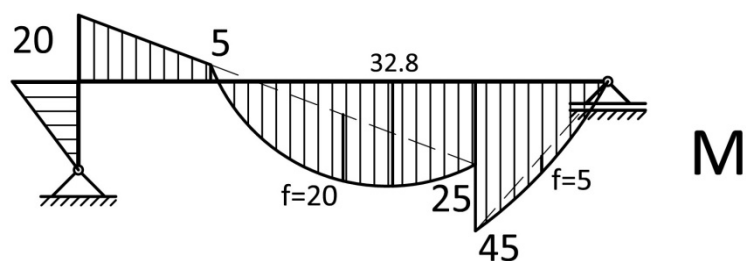
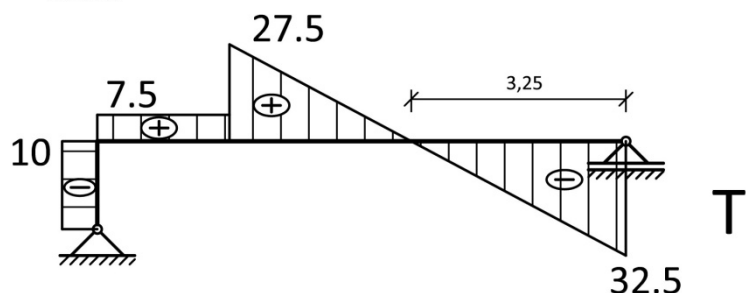
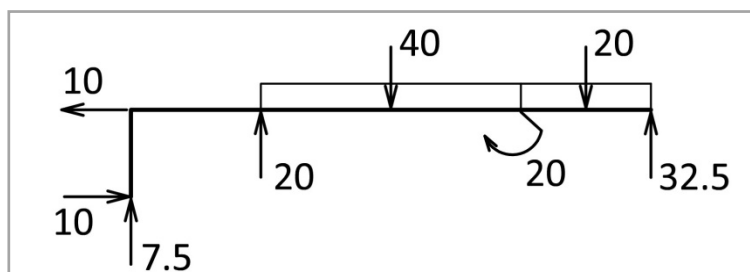
1. ЗАДАТАК (условни 50 %)

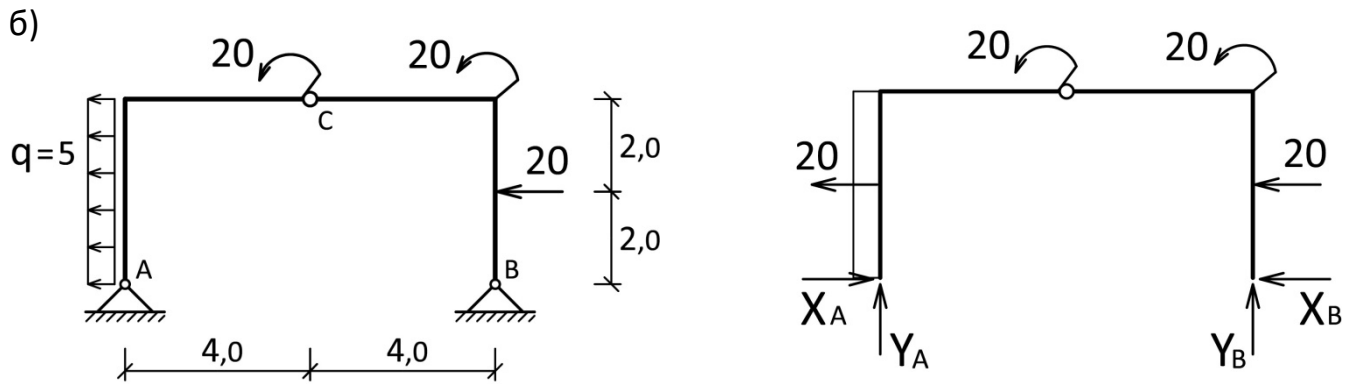


$$\sum F_x = 0 : X_A - 10 = 0 \rightarrow \underline{X_A = 10}$$

$$\sum M_A = 0 : Y_B \cdot 8 - 60 \cdot 5 - 20 + 20 \cdot 2 + 10 \cdot 2 = 0 \rightarrow \underline{Y_B = 32.5}$$

$$\sum F_y = 0 : Y_A + Y_B + 20 - 60 = 0 \rightarrow \underline{Y_A = 7.5}$$



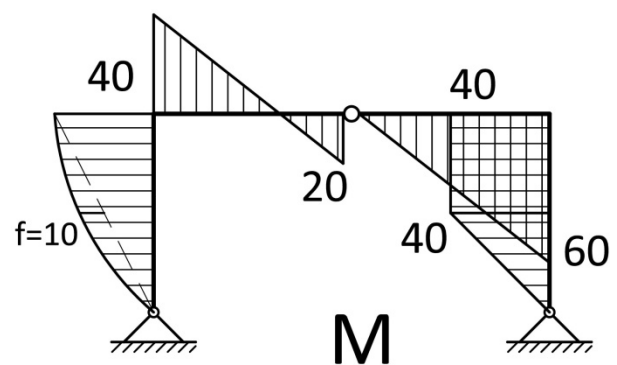
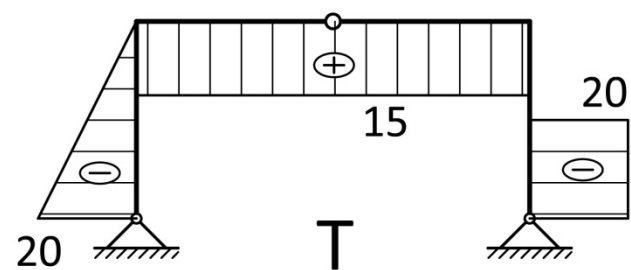
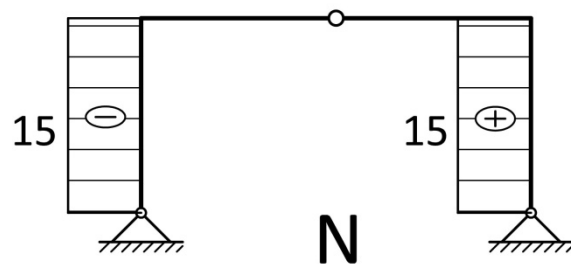
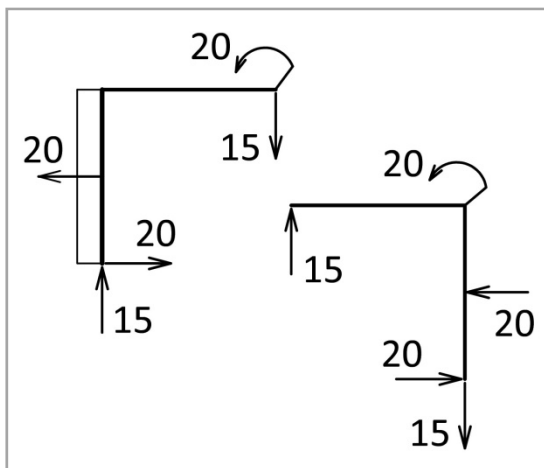


$$\sum M_A = 0 : Y_B \cdot 8 + 20 \cdot 2 + 20 + 20 + 20 \cdot 2 = 0 \rightarrow \underline{Y_B = -15}$$

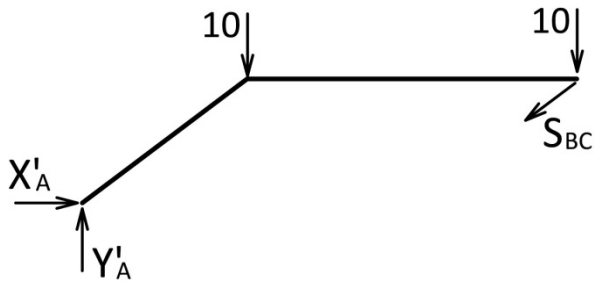
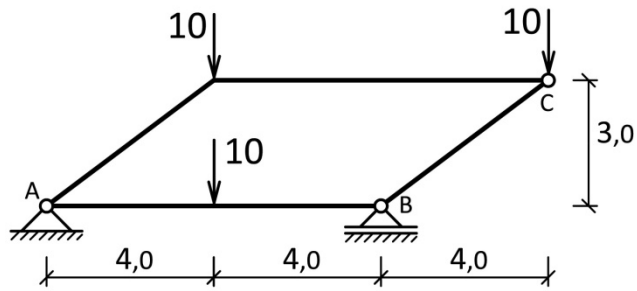
$$\sum F_Y = 0 : Y_A + Y_B = 0 \rightarrow \underline{Y_A = 15}$$

$$\sum M_{C, \text{dec}} = 0 : X_B \cdot 4 - Y_B \cdot 4 + 20 \cdot 2 - 20 = 0 \rightarrow \underline{X_B = -20}$$

$$\sum F_X = 0 : X_A - X_B - 20 - 20 = 0 \rightarrow \underline{X_A = 20}$$



B)

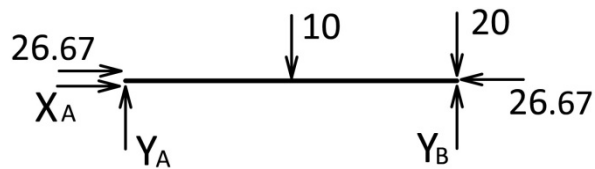


$$\sum M_A = 0 : 0.6S_{BC} \cdot 12 - 0.8S_{BC} \cdot 3 + 10 \cdot 12 + 10 \cdot 4 = 0$$

$$\rightarrow \underline{S_{BC} = -33.333} \quad (0.6S_{BC} = -20; 0.8S_{BC} = -26.667)$$

$$\sum F_X = 0 : X'_A - 0.8S_{BC} = 0 \rightarrow \underline{X'_A = -26.667}$$

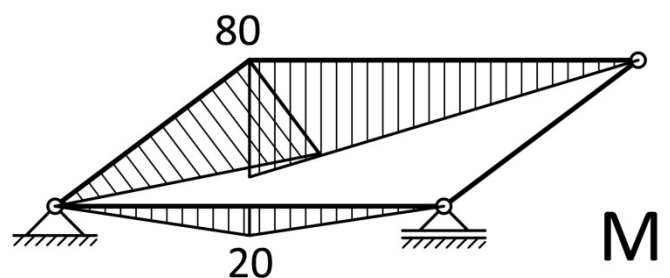
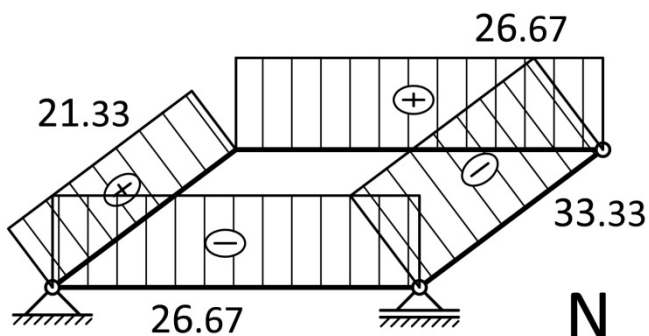
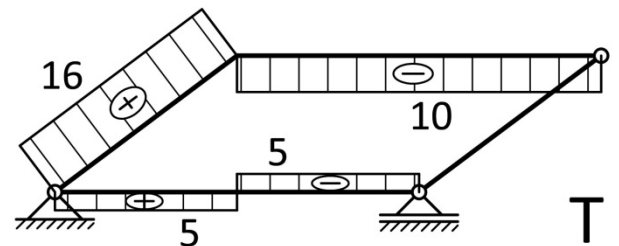
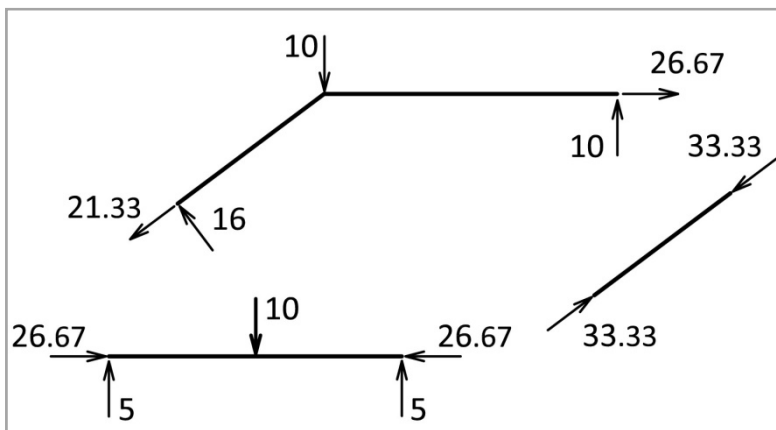
$$\sum F_Y = 0 : Y'_A - 0.6S_{BC} - 10 - 10 = 0 \rightarrow \underline{Y'_A = 0}$$



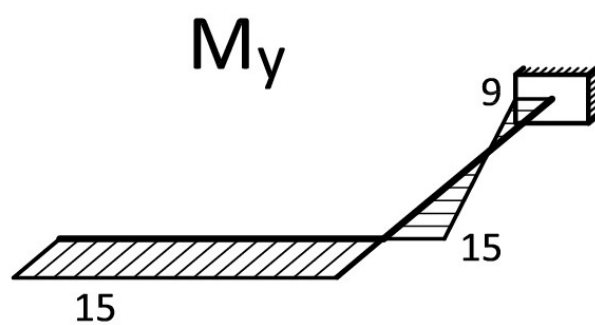
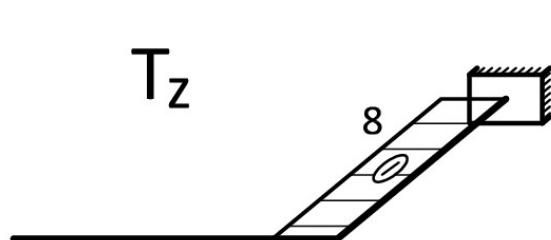
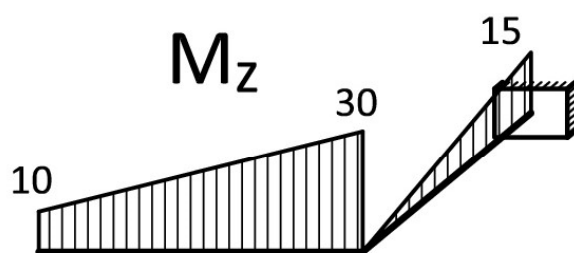
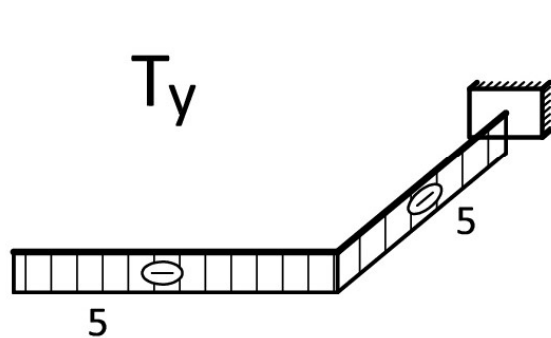
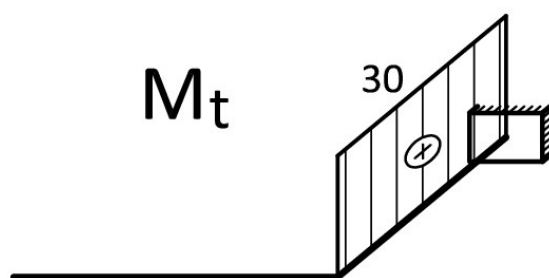
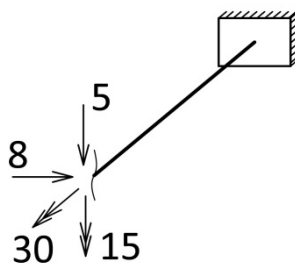
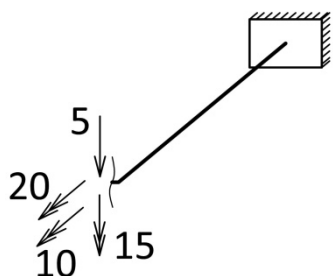
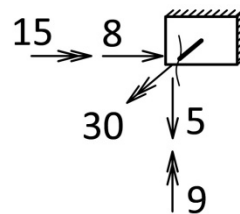
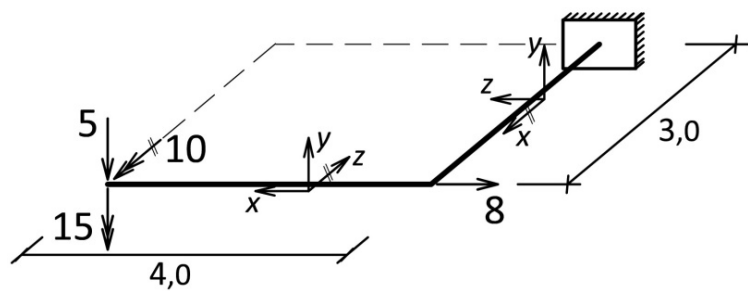
$$\sum F_X = 0 : X_A + 26.667 - 26.667 = 0 \rightarrow \underline{X_A = 0}$$

$$\sum M_A = 0 : Y_B \cdot 8 - 20 \cdot 8 - 10 \cdot 4 = 0 \rightarrow \underline{Y_B = 25}$$

$$\sum F_Y = 0 : Y_A + Y_B - 10 - 20 = 0 \rightarrow \underline{Y_A = 5}$$

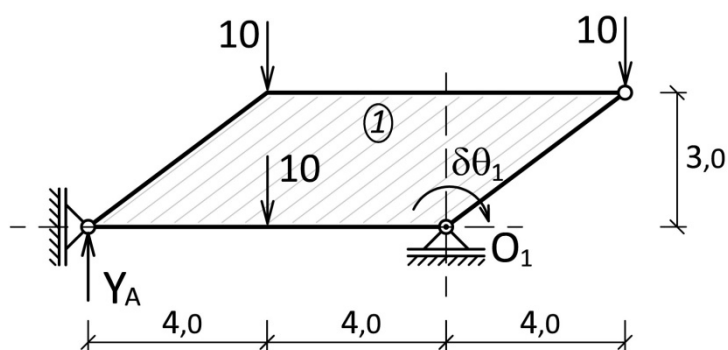


r)



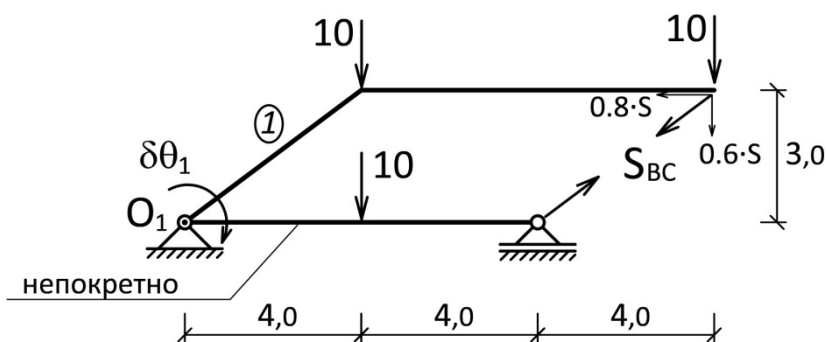
2. ЗАДАТАК (30 %)

а) $Y_A = ?$ (зад. 1в)



$$\delta A = Y_A \cdot (8 \cdot \delta \theta_1) - 10 \cdot (4 \cdot \delta \theta_1) - 10 \cdot (4 \cdot \delta \theta_1) + 10 \cdot (4 \cdot \delta \theta_1) = 0 \rightarrow \underline{Y_A = 5}$$

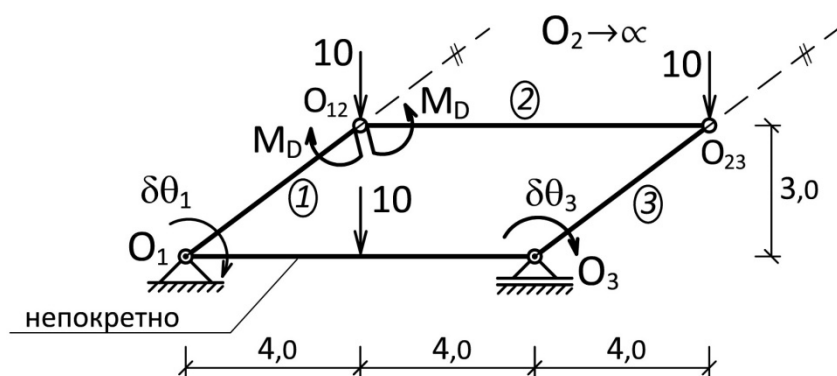
б) $S_{BC} = ?$ (зад. 1в)



$$\delta A = (0.6 \cdot S_{BC}) \cdot (12 \cdot \delta \theta_1) - (0.8 \cdot S_{BC}) \cdot (3 \cdot \delta \theta_1) + 10 \cdot (12 \cdot \delta \theta_1) + 10 \cdot (4 \cdot \delta \theta_1) = 0$$

$$4.8 \cdot S_{BC} \cdot \delta \theta_1 = -160 \cdot \delta \theta_1 \rightarrow \underline{S_{BC} = -33.333} \text{ (притиснут штап)}$$

в) $M_D = ?$ (зад. 1 в)



$$O_2 \rightarrow \infty \rightarrow \delta \theta_2 = 0$$

$$\delta r_{O12} = \delta \theta_1 \cdot 5 \rightarrow \delta \theta_1 = \delta \theta_3$$

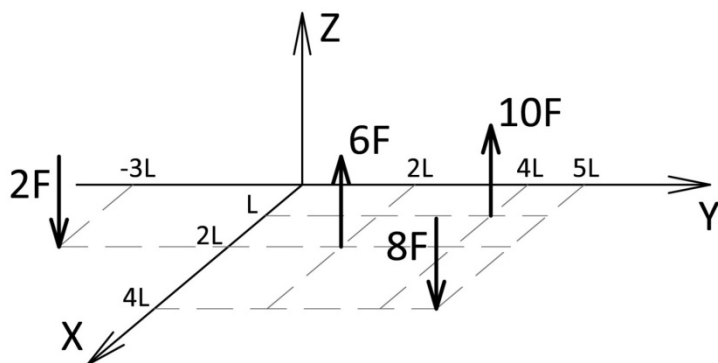
$$\delta r_{O23} = \delta \theta_3 \cdot 5$$

$$\delta A = M_D \cdot \delta \theta_1 + 10 \cdot (4 \cdot \delta \theta_1) + 10 \cdot (4 \cdot \delta \theta_3) = 0$$

$$M_D \cdot \delta \theta_1 + 80 \cdot \delta \theta_1 = 0 \rightarrow \underline{M_D = -80} \text{ (затегнута унутрашња страна)}$$

3. ЗАДАТАК (20 %)

б)



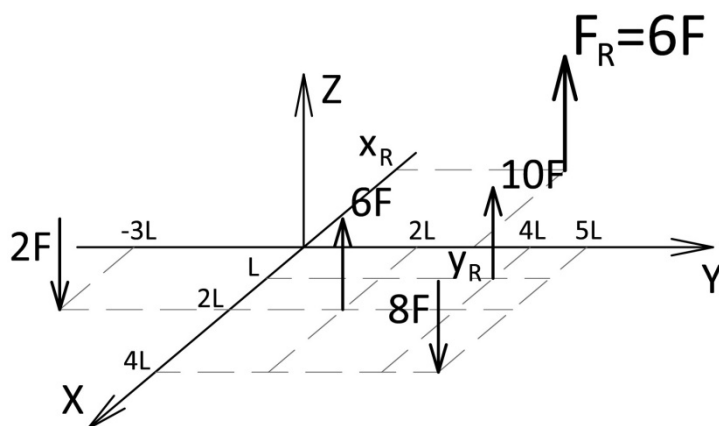
Вектор резултанте:

$$F_{RZ} = \sum F_z = 10F + 6F - 8F - 2F = 6F$$

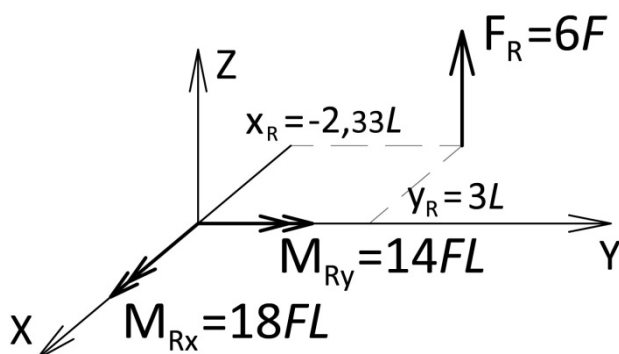
Продорна тачка нападне линије резултанте кроз x-y раван:

$$x_R = \frac{\sum x_i \cdot F_i}{F_{RZ}} = \frac{10F \cdot L + 6F \cdot 2L + (-8F) \cdot 4L + (-2F) \cdot 2L}{6F} = -2.33L$$

$$y_R = \frac{\sum y_i \cdot F_i}{F_{RZ}} = \frac{10F \cdot 4L + 6F \cdot 2L + (-8F) \cdot 5L + (-2F) \cdot (-3L)}{6F} = 3L$$



в) Редукциони моменат резултанте у односу на коорд.почетак:



$$M_{Rx}^{(0)} = 6F \cdot 3L = 18FL$$

$$M_{Ry}^{(0)} = 6F \cdot 2.33L = 14FL$$