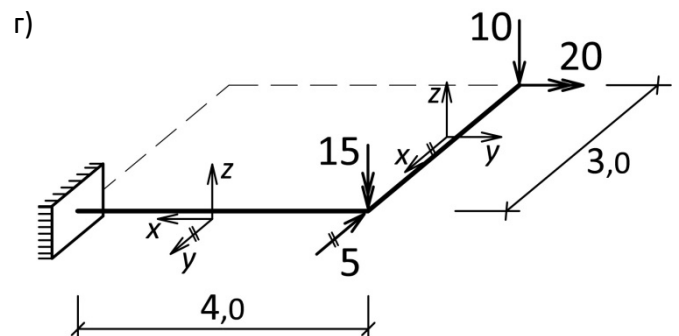
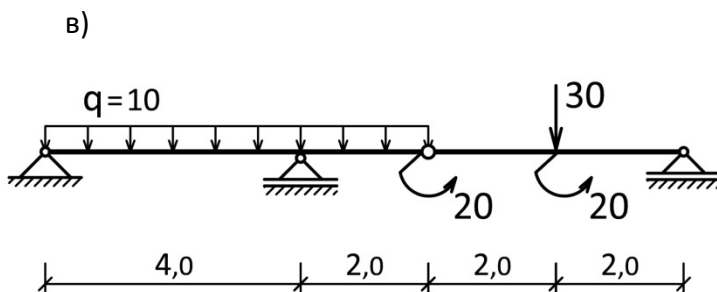
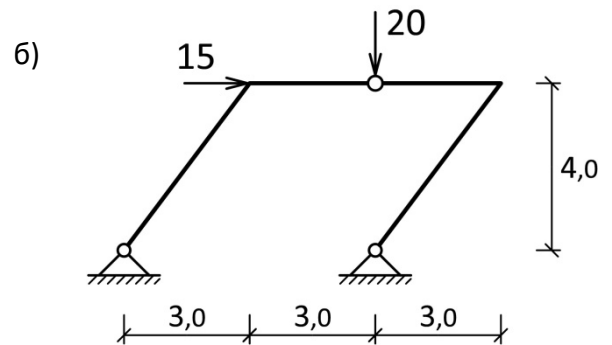
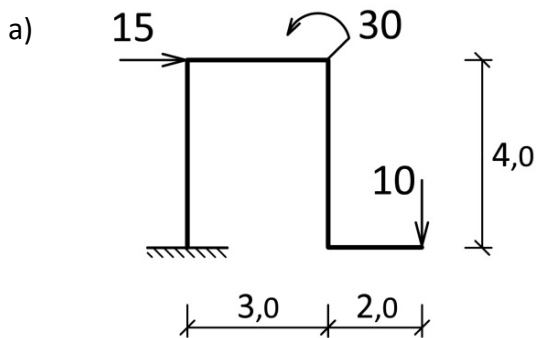


ГРАЂЕВИНСКИ ФАКУЛТЕТ УНИВЕРЗИТЕТА У БЕОГРАДУ
 Усмени (теоријски) део испита из **ТЕХНИЧКЕ МЕХАНИКЕ 1**
 (писмени део одржан 24.06.2020.)

1. ЗАДАТАК (условни 50 %)

Нацртати дијаграме сила у пресеку за приказане носаче.

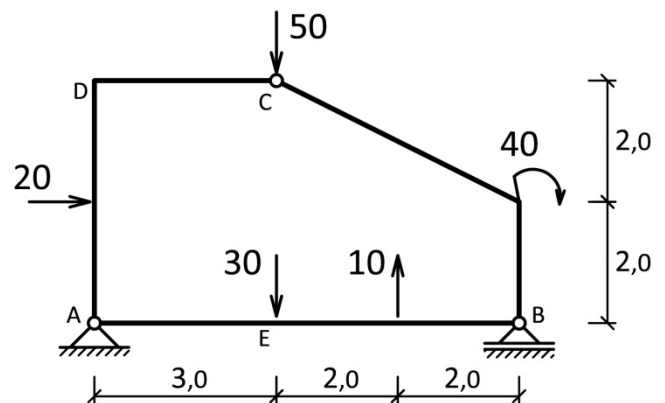


2. ЗАДАТАК (26 %)

а) Написати израз за виртуелни рад сила које делују на слободно круто тело.

б) Применом опште једначине статике, код приказаног носача одредити:

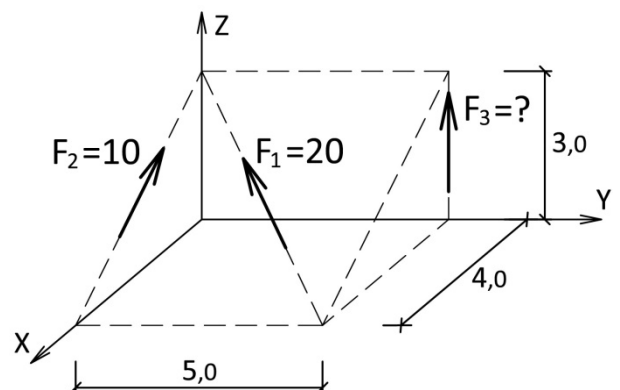
- * реакцију везе у ослоњу В,
- * моменат савијања у крутом углу D,
- * моменат савијања у тачки Е.



3. ЗАДАТАК (24 %)

а) Дефинисати моменат силе за осу.

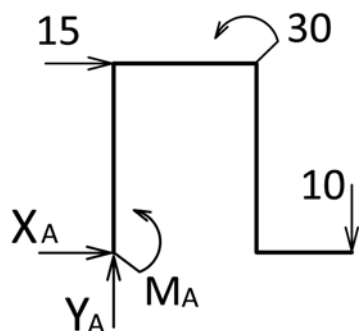
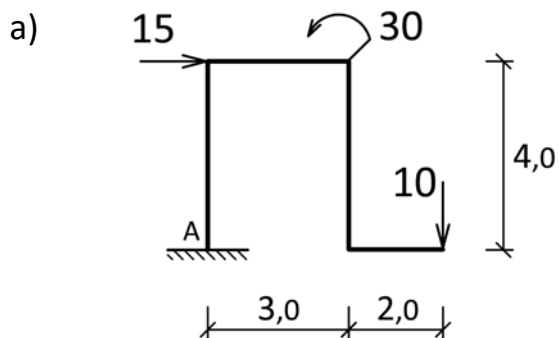
б) Приказани систем сила се састоји од две познате и једне непознате силе. Одредити непознату силу ($F_3=?$) тако да систем има резултанту, а затим наћи вектор резултанте и једначину њене нападне линије.



Напомена: У свим задацима димензије за дужине и силе су: m, kN

- Р Е Ш Е Њ А -

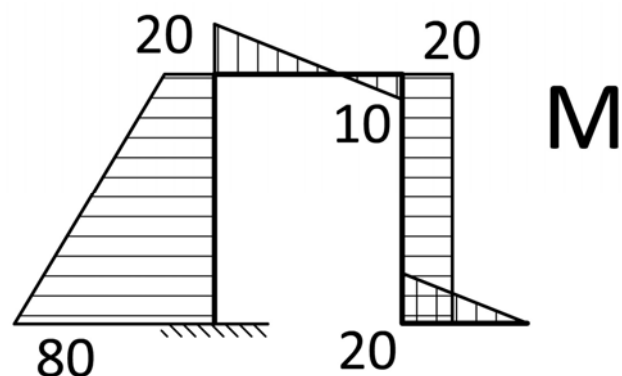
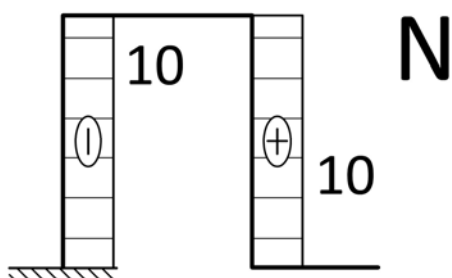
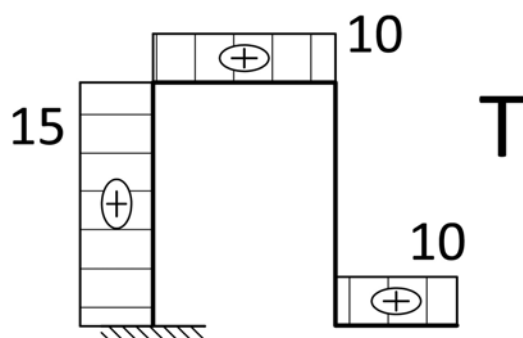
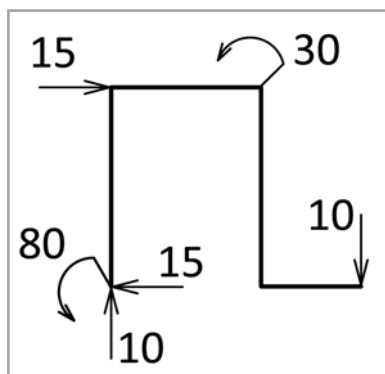
1. ЗАДАТАК (условни 50 %)



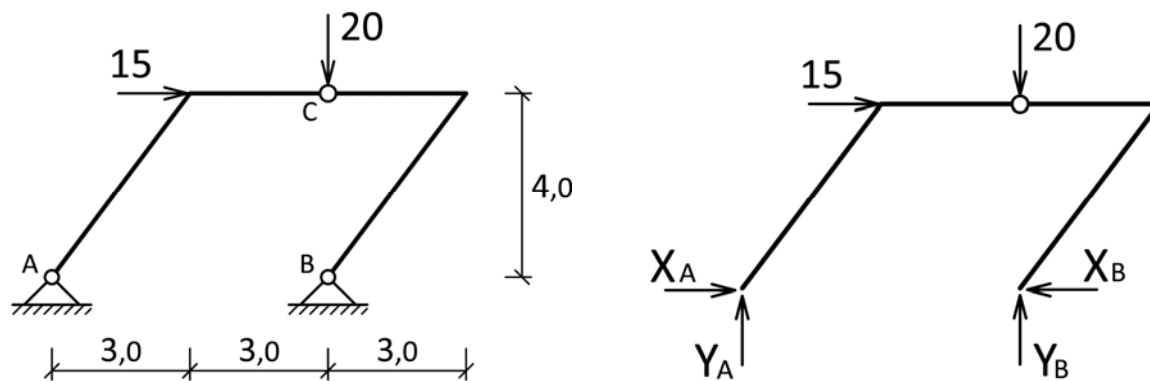
$$\sum F_X = 0 : X_A + 15 = 0 \rightarrow \underline{X_A = -15}$$

$$\sum F_Y = 0 : Y_A - 10 = 0 \rightarrow \underline{Y_A = 10}$$

$$\sum M_A = 0 : M_A - 15 \cdot 4 - 10 \cdot 5 + 30 = 0 \rightarrow \underline{M_A = 80}$$



6)

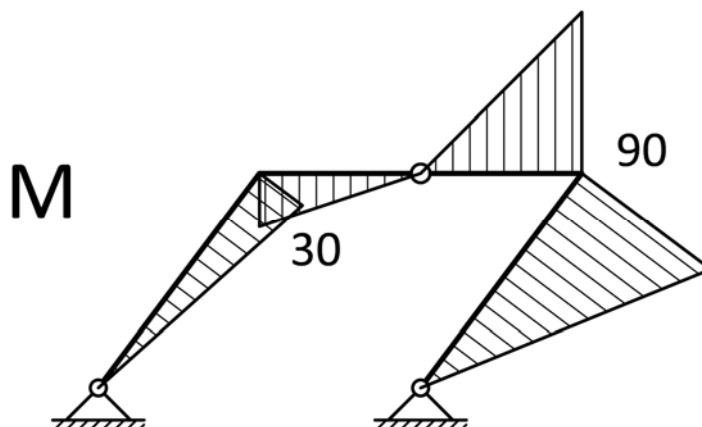
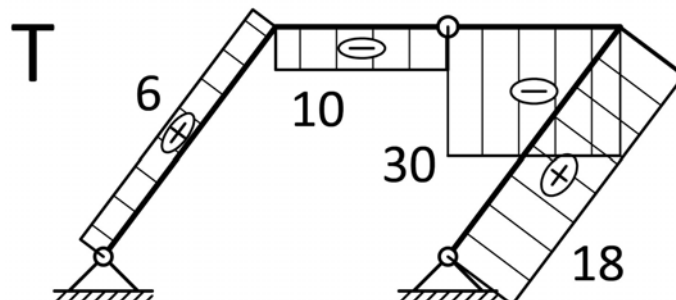
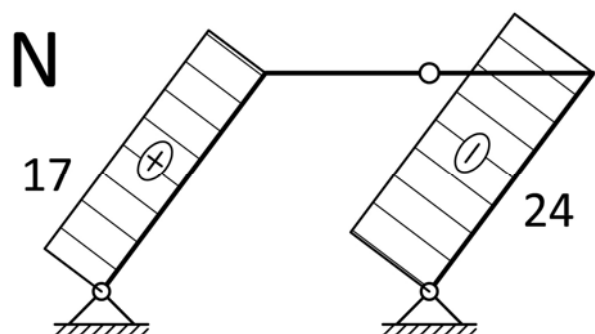
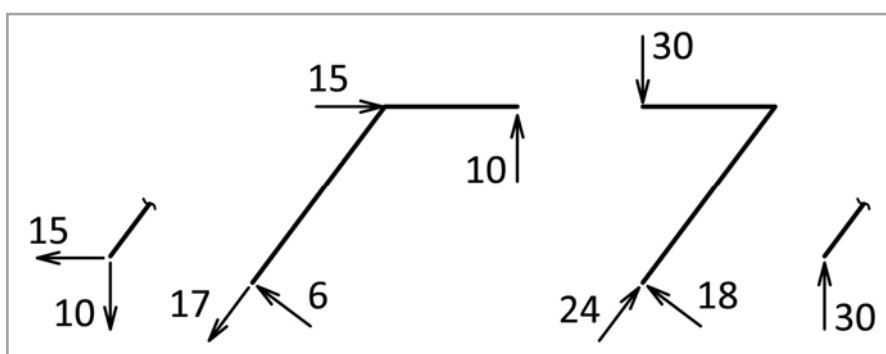


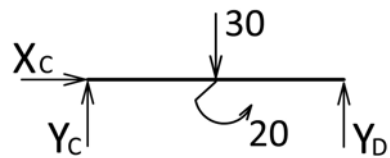
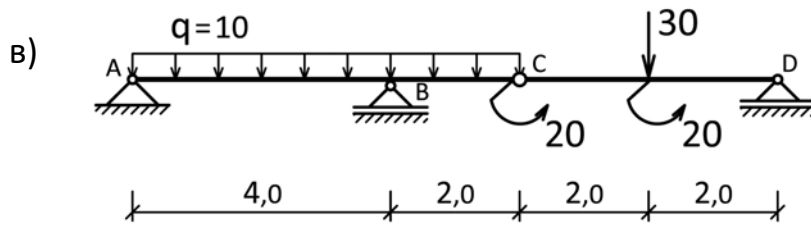
$$\sum M_A = 0 : Y_B \cdot 6 - 20 \cdot 6 - 15 \cdot 4 = 0 \rightarrow \underline{Y_B = 30}$$

$$\sum F_Y = 0 : Y_A + Y_B - 20 = 0 \rightarrow \underline{Y_A = -10}$$

$$\sum M_{C,dec} = 0 : X_B \cdot 4 = 0 \rightarrow \underline{X_B = 0}$$

$$\sum F_X = 0 : X_A - X_B + 15 = 0 \rightarrow \underline{X_A = -15}$$

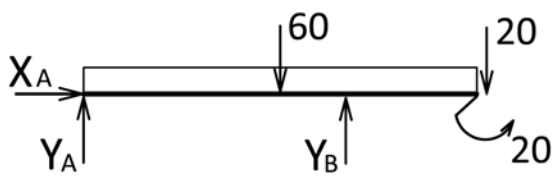




$$\sum F_x = 0 : \underline{X_B = 0}$$

$$\sum M_C = 0 : Y_D \cdot 4 - 30 \cdot 2 + 20 = 0 \rightarrow \underline{Y_D = 10}$$

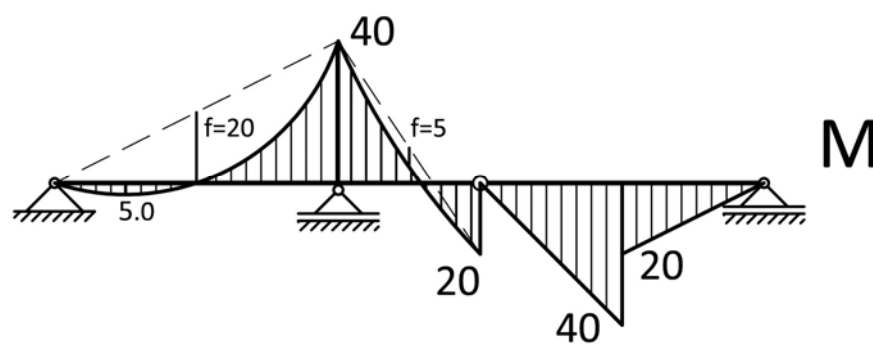
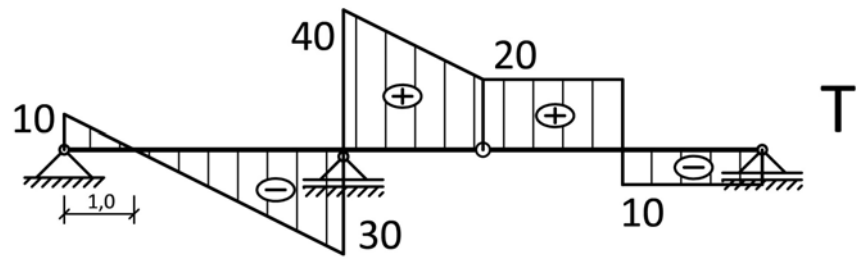
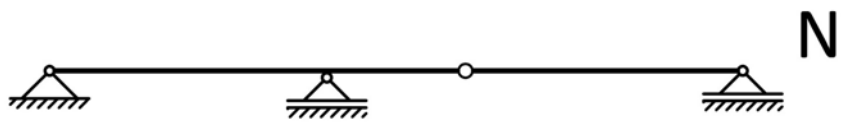
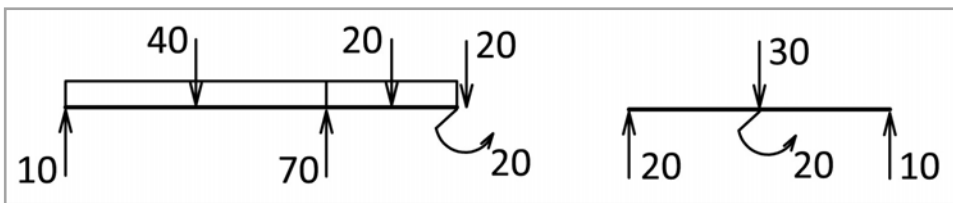
$$\sum F_y = 0 : Y_C + Y_D - 30 = 0 \rightarrow \underline{Y_C = 20}$$



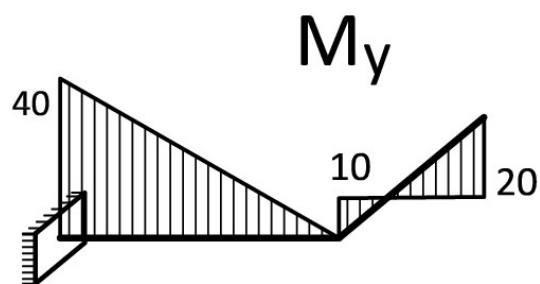
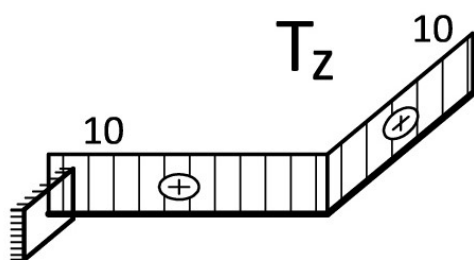
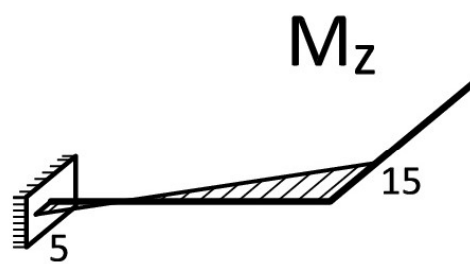
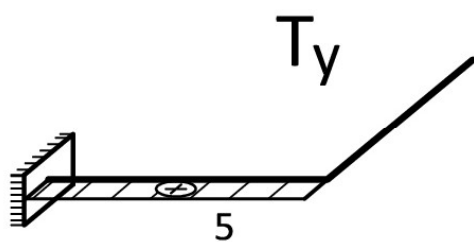
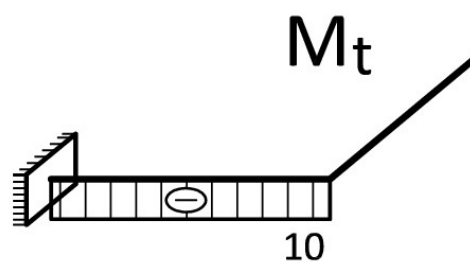
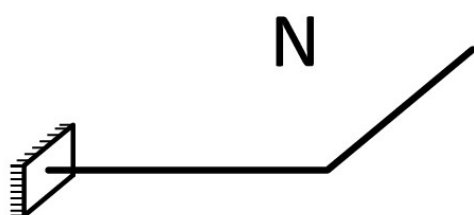
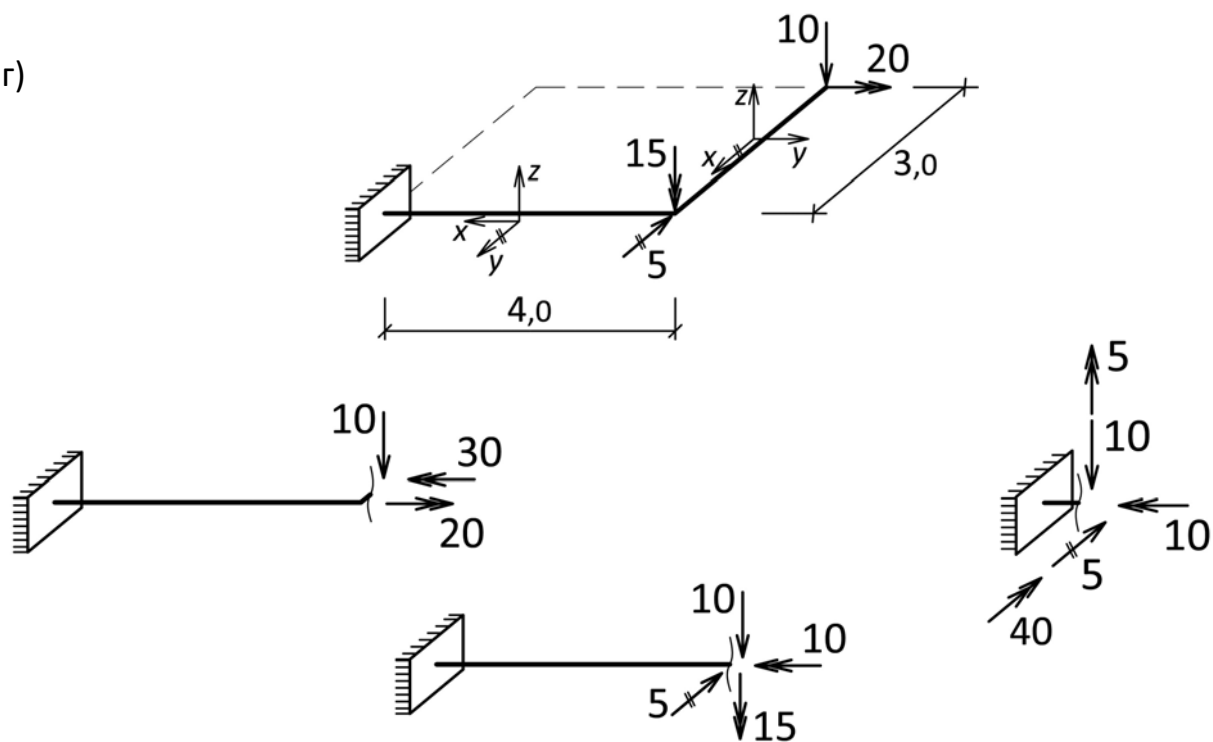
$$\sum F_x = 0 : \underline{X_A = 0}$$

$$\sum M_D = 0 : Y_B \cdot 4 - 20 \cdot 6 - 60 \cdot 3 + 20 = 0 \rightarrow \underline{Y_B = 70}$$

$$\sum F_y = 0 : Y_A + Y_B - 20 - 60 = 0 \rightarrow \underline{Y_A = 10}$$

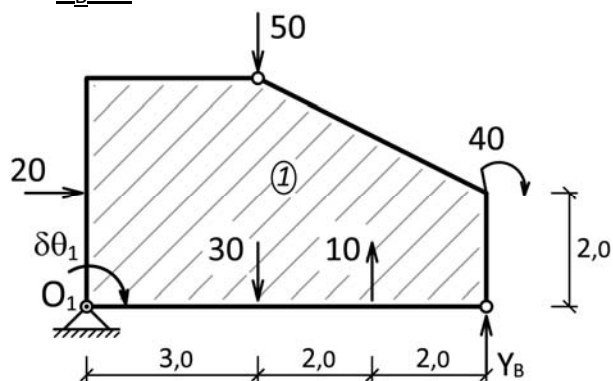


r)



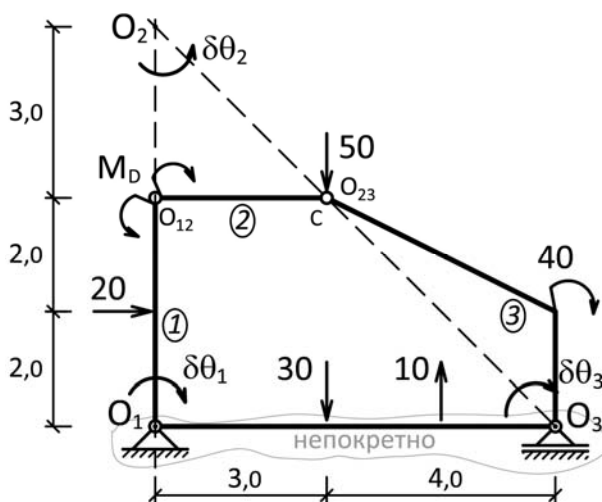
2. ЗАДАТАК (26 %)

б) $Y_B = ?$



$$\begin{aligned}\delta A &= -Y_B \cdot (7 \cdot \delta \theta_1) - 10 \cdot (5 \cdot \delta \theta_1) + 30 \cdot (3 \cdot \delta \theta_1) \\ &\quad + 20 \cdot (2 \cdot \delta \theta_1) + 50 \cdot (3 \cdot \delta \theta_1) + 40 \cdot \delta \theta_1 = 0 \\ &\Rightarrow \underline{Y_B = 38.57}\end{aligned}$$

$M_D = ?$

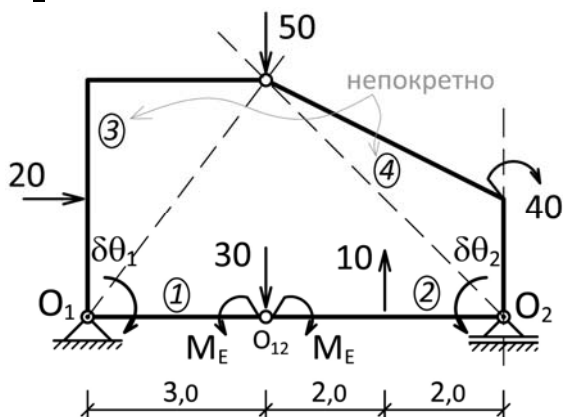


$$\delta r_{O12} = \delta \theta_1 \cdot 4 = \delta \theta_2 \cdot 3 \rightarrow \delta \theta_1 = 0.75 \cdot \delta \theta_2$$

$$\delta r_{O23,y} = \delta \theta_2 \cdot 3 = \delta \theta_3 \cdot 4 \rightarrow \delta \theta_3 = 0.75 \cdot \delta \theta_2$$

$$\begin{aligned}\delta A &= 20 \cdot (2 \cdot \delta \theta_1) - M_D \cdot \delta \theta_1 - M_D \cdot \delta \theta_2 - 50 \cdot (3 \cdot \delta \theta_2) + 40 \cdot \delta \theta_3 = 0 \\ 1.75 \cdot M_D \cdot \delta \theta_2 + 90 \cdot \delta \theta_2 &= 0 \quad \Rightarrow \underline{M_D = -51.43}\end{aligned}$$

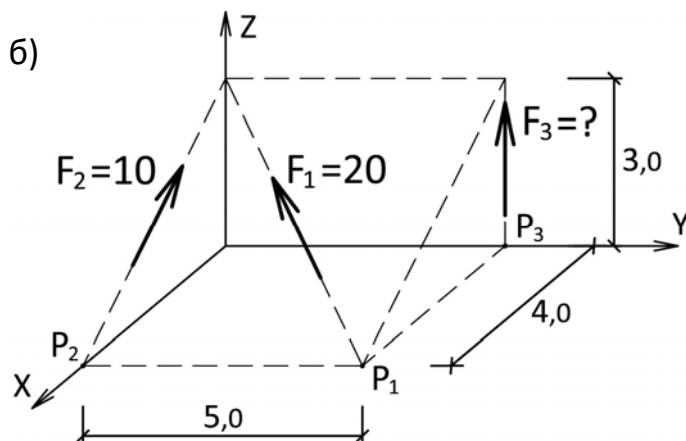
$M_E = ?$



$$\delta r_{O12} = \delta \theta_1 \cdot 3 = \delta \theta_2 \cdot 4 \rightarrow \delta \theta_2 = 0.75 \cdot \delta \theta_1$$

$$\begin{aligned}\delta A &= -M_E \cdot \delta \theta_1 + 30 \cdot (3 \cdot \delta \theta_1) \\ &\quad - M_E \cdot \delta \theta_2 - 10 \cdot (2 \cdot \delta \theta_1) = 0 \\ &\Rightarrow \underline{M_E = 42.86}\end{aligned}$$

3. ЗАДАТАК (24 %)



$$\vec{F}_1 = \left\{ -20 \cdot \frac{4}{5\sqrt{2}}, -20 \cdot \frac{5}{5\sqrt{2}}, 20 \cdot \frac{3}{5\sqrt{2}} \right\}$$

$$\vec{F}_2 = \left\{ -10 \cdot \frac{4}{5}, 0, 10 \cdot \frac{3}{5} \right\}$$

$$\vec{F}_3 = \{ 0, 0, F_3 \}$$

$$\vec{F}_R = \{ -19.31, -14.14, F_3 + 14.48 \}$$

$$\vec{M}_1^{(o)} = \overrightarrow{OP_1} \times \vec{F}_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 5 & 0 \\ -8\sqrt{2} & -10\sqrt{2} & 6\sqrt{2} \end{vmatrix} = \{ 42.43, -33.94, 0 \}$$

$$\vec{M}_2^{(o)} = \overrightarrow{OP_2} \times \vec{F}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 0 & 0 \\ -8 & 0 & 6 \end{vmatrix} = \{ 0, -24, 0 \}$$

$$\vec{M}_3^{(o)} = \overrightarrow{OP_3} \times \vec{F}_3 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & F_3 \\ 0 & 5 & 0 \end{vmatrix} = \{ 5F_3, 0, 0 \}$$

$$\vec{M}_R^{(o)} = \{ 5F_3 + 42.43, -57.94, 0 \}$$

Услов за постојање РЕЗУЛТАНТЕ: $\vec{F}_R \cdot \vec{M}_R = 0$

$$(-19.31) \cdot (5F_3 + 42.43) + (-14.14) \cdot (-57.94) + 0 = 0 \quad \rightarrow \quad F_3 = 0$$

$$\Rightarrow \vec{F}_R = \{ -19.31, -14.14, 14.48 \}, \quad |\vec{F}_R| = 27.97 \quad \Rightarrow \vec{M}_R^{(o)} = \{ 42.43, -57.94, 0 \}$$

Једначина нападне линије резултанте:

$$\frac{x + \frac{M_{RY}}{Z_R}}{X_R} = \frac{y - \frac{M_{RX}}{Z_R}}{Y_R} = \frac{z}{Z_R} \quad \Rightarrow \quad \frac{x + \frac{(-57.94)}{14.48}}{-19.31} = \frac{y - \frac{42.43}{14.48}}{-14.14} = \frac{z}{14.48}$$

$$\Rightarrow \begin{cases} x + 1.33z - 4 = 0 \\ y + 0.98z - 2.93 = 0 \end{cases}$$