REŠAVANJE DVA PUTA STATIČKI NEODREĐENOG NOSAČA (REŠEN BROJNI PRIMER)

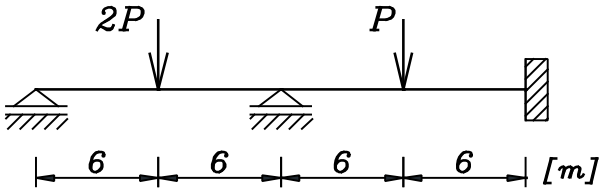
Za nosač i opterećenje prikazano na slici, treba:

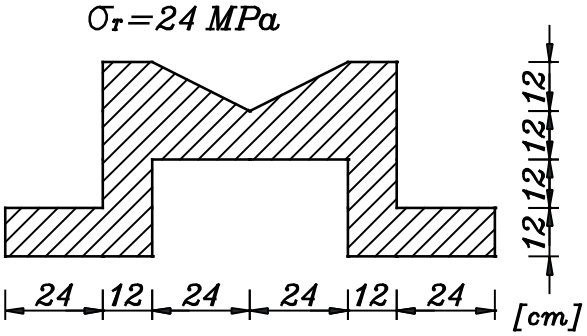
a) Nacrtati sve moguće mehanizme loma.

b) Kinematičkom metodom odrediti graničnu vrednost parametra opterećenja P^* (u funkciji od M^*).

c) Nacrtati dijagram momenata (u funkciji od M^*) za $P = 0.5 \cdot P^*$ i $P = 0.95 \cdot P^*$.

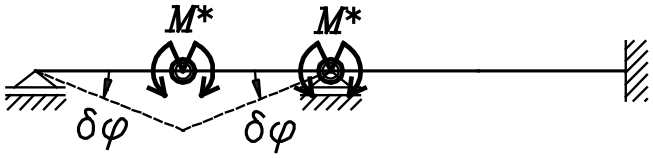
d) Za zadati poprečni presek odrediti M^* .





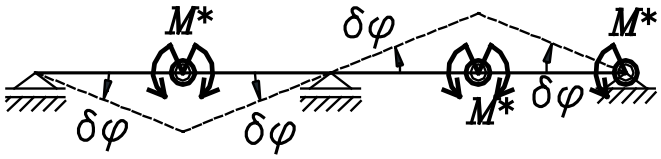
REŠENJE:

a)

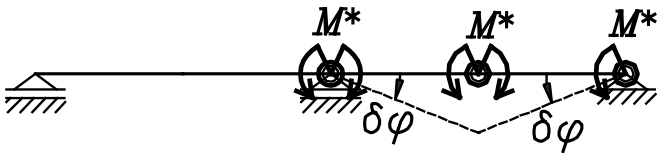


b)

$$3M^* \cdot \delta\varphi = 2P_1^I \cdot 6\delta\varphi \rightarrow \; P_1^I = 0.25M^*$$



$$5M^* \cdot \delta\varphi = 2P_1^{II} \cdot 6\delta\varphi - P_1^{II} \cdot 6\delta\varphi \rightarrow \; P_1^{II} = 0.83M^*$$

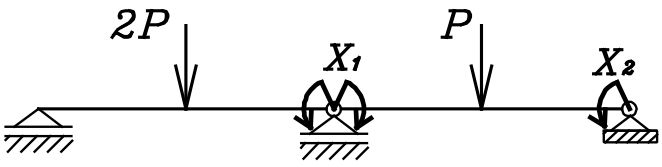


$$4M^* \cdot \delta\varphi = P_1^{III} \cdot 6\delta\varphi \rightarrow \; P_1^{III} = 0.67M^*$$

$$P^* = \min\{P_1^I, P_1^{II}, P_1^{III}\} = 0.25M^*$$

c) Rešavanje dva puta statički neodređenog nosača

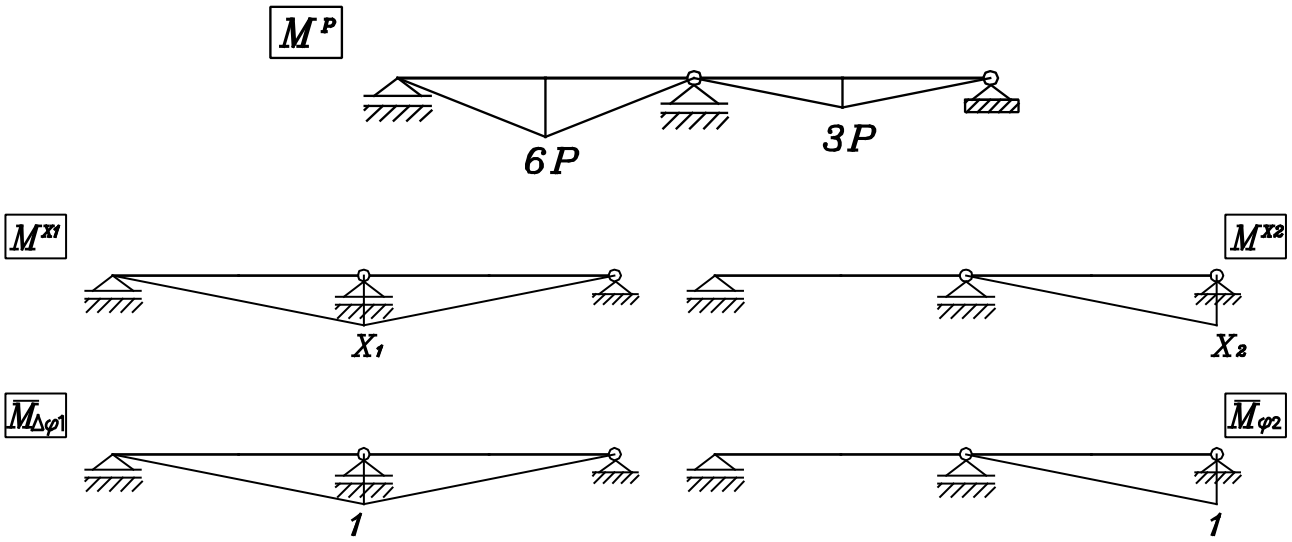
Osnovni sistem i statički nepoznate:



Geometrijski uslov:

$\Delta\varphi_1 = \Delta\varphi_1^P + \Delta\varphi_1^{X_1} + \Delta\varphi_1^{X_2} = 0$

$\varphi_2 = \varphi_2^P + \varphi_2^{X_1} + \varphi_2^{X_2} = 0$



$EI\Delta\varphi_1^P = \int M^{(P)} \cdot \overline{M_{\Delta\varphi_1}}\, ds = \frac{6}{3} \cdot 6P \cdot 0.5 + \frac{6}{6} \cdot 6P \cdot (2 \cdot 0.5 + 1) + \frac{6}{6} \cdot 3P \cdot (2 \cdot 0.5 + 1) + \frac{6}{3} \cdot 3P \cdot 0.5 = 27P$

$EI\varphi_2^P = \int M^{(P)} \cdot \overline{M_{\varphi_2}}\, ds = \frac{6}{3} \cdot 3P \cdot 0.5 + \frac{6}{3} \cdot 3P \cdot (2 \cdot 0.5 + 1) = 9P$

$EI\Delta\varphi_1^{X_1} = \int M_{X1} \cdot \overline{M_{\Delta\varphi_1}}\, ds = \frac{12}{3} \cdot 1 \cdot X_1 \cdot 2 = 8X_1$

$EI\Delta\varphi_1^{X_2} = \frac{12}{6} \cdot 1 \cdot X_2 = 2X_2$

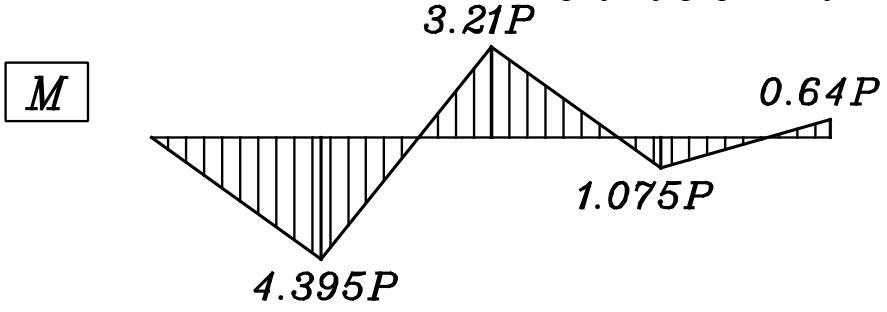
$EI\varphi_2^{X_1} = \frac{12}{6} \cdot 1 \cdot X_1 = 2X_1$

$EI\varphi_2^{X_2} = \frac{12}{3} \cdot 1 \cdot X_2 = 4X_2$

$\left.\begin{matrix} 27P + 8X_1 + 2X_2 = 0 \\ 9P + 2X_1 + 4X_2 = 0 \end{matrix}\right\} \rightarrow \left.\begin{matrix} 8X_1 + 2X_2 = -27P \\ 2X_1 + 4X_2 = -9P \end{matrix}\right\} \rightarrow \begin{matrix} X_1 = -3.21P \\ X_2 = -0.64P \end{matrix}$

$\rightarrow X_1 = -3.21P\ ;\ X_2 = -0.64P$

Dijagram momenta na statički neodređenom nosaču usled spoljašnjeg opterećenja:



- Formiranje **prvog** plastičnog zgloba:

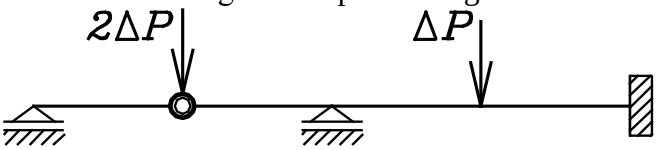
$M_{max} = 4.395P_1 = M^*$

$P_1 = 0.2275M^*$ odnosno, $P_1 = 0.91P^*$

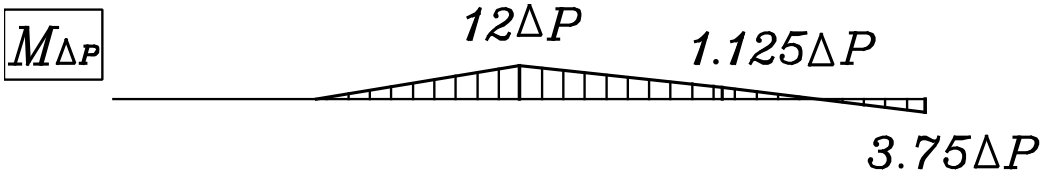
Dakle, prvi plastični zglob se formira na mestu delovanja sile 2P.

- Formiranje **drugog** plastičnog zgloba:

Rešavanjem jedanput statički neodređenog nosača prikazanog na slici



dobijamo sledeći dijagram momenta savijanja usled dodatnog opterećenja ΔP:



Drugi plastični zglob će se formirati na mestu gde se dostiže maksimum

$M_{max} = max(M(P_1) + M(\Delta P)) = M^* \ tj. \ 3.21P_1 + 12\Delta P = M^*$

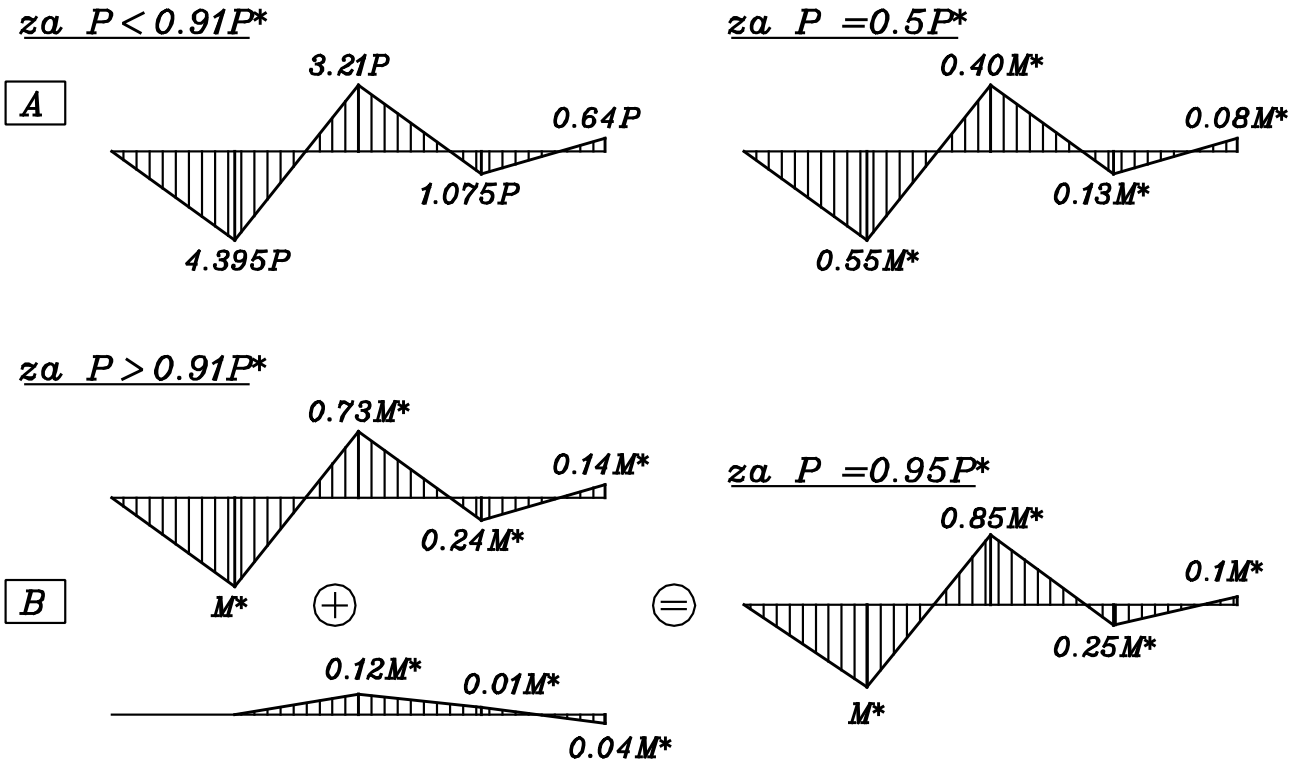
$3.21 \cdot 0.2275M^* + 12\Delta P = M^* \ \rightarrow \Delta P = 0.0225M^*$

$P^* = P_1 + \Delta P = 0.2275M^* + 0.0225M^* = 0.25M^*$

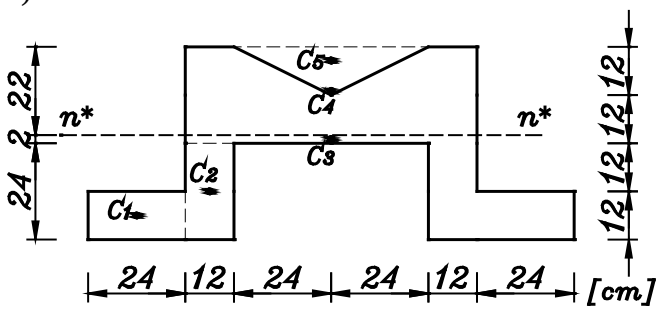
Dakle, drugi plastični zglob će se formirati iznad srednjeg oslonca pri ΔP=0.0225 M* , odnosno za P=P_1+ ΔP= 0.25 M*, kako je dobijeno i kinematičkom metodom.

$\frac{P_1}{P^*} = \frac{0.2275M^*}{0.25M^*} = 0.91M^*$

Traženi dijagrami momenta:



d)



$z_1 = 20\ cm$
$z_2 = 14\ cm$
$z_3 = 1\ cm$
$z_4 = 11\ cm$
$z_5 = 18\ cm$

$$A = 24 \cdot 12 \cdot 2 + 24 \cdot 12 \cdot 2 + 72 \cdot 24 - \frac{48 \cdot 12}{2} = 576 + 576 + 1728 - 288 = 2592cm^2$$

$$\frac{A}{2} = 1296cm^2$$

$1296 - 576 \cdot 2 = 144cm^2$ $\qquad \frac{144}{72} = 2cm$

$S_{pl} = 576 \cdot 20 + 576 \cdot 14 + 72 \cdot 2 \cdot 1 + 72 \cdot 22 \cdot 11 - 288 \cdot 18 = 11520 + 8064 + 144 +$

$17424 - 5184 = 31968cm^3$

$M^* = S_{pl} \cdot \sigma_T = 31968 \cdot 10^{-6} \cdot 24 \cdot 10^6 = 767.232kNm$